Prof. U. Walter
Institute of Astronautics
TU Munich

Research question

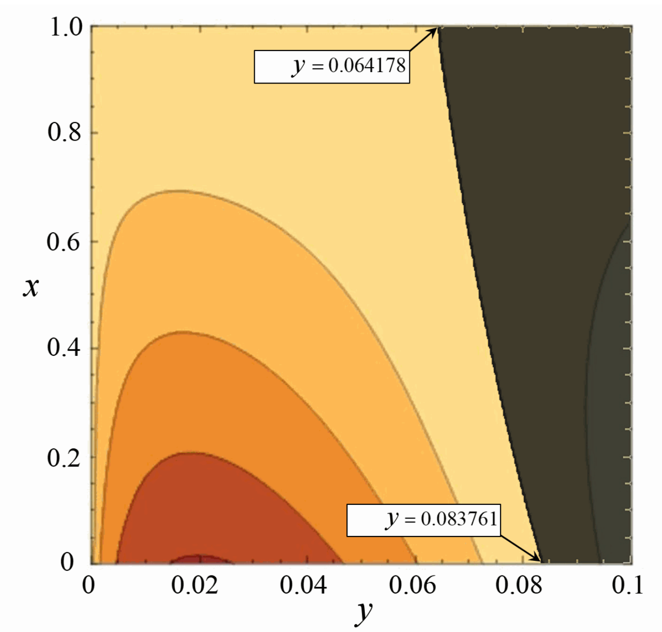
This is a mathematical problem from space orbital mechanics. The following equation

$$\Delta v_{bi} = \left(\sqrt{\frac{2}{1+xy}}-1\right)+\sqrt{y}\left(\sqrt{\frac{2}{1+x}}-1\right)+\sqrt{2xy}\left(\sqrt{\frac{1}{1+1/x}}-\sqrt{\frac{1}{1+1/(xy)}}\right)-\left(1-\sqrt{y}\right)\left(\frac{1+\sqrt{y}}{\sqrt{(1+y)/2}}-1\right)$$

is the normalized difference between the transition effort of a so-called bi-elliptic transfer between an inner and an outer circular orbit with radius $a_{\bullet}$ and $a_{\square}$, respectively, via an interim orbit with radius $r_{\times}$ (first three terms) and a direct Hohmann transfer (last term). Here

$$y \equiv a_{\bullet}/a_{\square} \leq 1$$
$$x \equiv a_{\square}/r_{\times} \leq 1$$

A contour plot of the function delivers the shown figure, where the bright part indicates where the bi-elliptic transfer is more efficient. Of practical importance is the borderline to the dark area where $\Delta v_{bi} = 0$.



As can be verified by insertion, for $x = 1$, $\Delta v_{bi} = 0$ delivers $y = 0$. However, it turns out that for the limit $x \to 1$ the end-border-point is $y \approx 0.0641778$ and hard to determine more precisely. So, the **problem statement reads**:

For the limit $x \to 1$, what is the y to at least 10 decimal places that satisfies $\Delta v_{bi}(x \to 1, y) = 0$

Possibly show that no such limit $y > 0$ exists.